RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2019

FIRST YEAR (BATCH 2018-21)

Date : 16/05/2019 **MATHEMATICS (Honours)**

Time : 11.00 am – 3.00 pm **Paper : II** Full Marks : 100

[Use a separate Answer Book for each Group]

Group - A

Answer any five questions from Question Nos. 1 to 8: $[5\times5]$

1. If $(1+x)^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ (n be a positive integer), prove that

$$a_0 + a_4 + a_8 + ... = 2^{n-2} + 2^{\frac{n}{2}-1}\cos\frac{n\pi}{4}.$$

2. Find the general values and the principal value of $(-1)^{\sqrt{2}}$. (3+2)

3. a) If a, b, c be positive real numbers such that the sum of any two is greater than the third, then prove that $a^2yz + b^2zx + c^2xy \le 0$ for all real x, y, z such that $x+y+z=0$. (3)

   b) If n be a positive integer >1, prove that $\left(\frac{2n+1}{3}\right)^{\frac{n(n+1)}{2}} > 1\cdot 2^2 \cdot 3^3 \cdot ..... \cdot n^n > \left(\frac{n+1}{2}\right)^{\frac{n(n+1)}{2}}$. (2)

4. a) Show that the maximum value of $(4-x)^3(2+x)^6$ is 2^{15}. (2)

   b) Show that the minimum value of $\frac{(5+x)(14+x)}{2+x}$ is 27. (3)

5. Solve the equation $x^4 - x^3 + 2x^2 - x + 1 = 0$ which has four distinct roots of equal moduli .

6. If α, β, γ are the roots of $x^3 + qx + r = 0$ $(r \ne 0)$, find the value of

   i) $\sum\frac{\alpha^2}{\beta\gamma}$ and ii) $\sum\frac{1}{(\alpha+\beta)^2}$.

7. Solve the equation $x^5 - 1 = 0$ .Deduce the values of $\cos\frac{\pi}{5}$ and $\cos\frac{2\pi}{5}$.

8. Solve the equation: $x^3 - 15x^2 - 33x + 847 = 0$ by Cardan's method .

Answer any five questions from Question Nos. 9 to 16: $[5\times5]$

9. a) Let $\{K_n\}_n$ be a sequence of non empty compact sets in $\mathbb{R}$ such that $K_1 \supseteq K_2 \supseteq K_3 \supseteq .... \supseteq K_n \supseteq ....$. Prove that there exist at least one point $x \in \mathbb{R}$ such that $x \in K_n\ \forall n$. (4)

   b) Find an infinite collection $\{K_n\}_n$ of compact sets in $\mathbb{R}$ such that the union $\bigcup_{n=1}^{\infty} K_n$ is not compact . (1)

10. a) Show that $[2,4]\setminus\{3\}$ is not compact . (2)

    b) If K is a compact subset of $\mathbb{R}$ then show that K is bounded . (3)

11. Let $f:[a,b]\to\mathbb{R}$ be a continuous function. Let $g:[a,b]\to\mathbb{R}$ be defined by $g(a)=f(a)$ and $g(x)=\max\{f(t):t\in[a,x]\}$ for $a<x\le b$. Show that g is continuous on [a, b] .

12. a) If $f:A\to\mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if $\{x_n\}_n$ is a Cauchy sequence in A , then show that $\{f(x_n)\}_n$ is a Cauchy sequence in $\mathbb{R}$. (3)

b) If $f:A\to\mathbb{R}$ is a Lipschitz function, then show that f is uniformly continuous on A. (2)

13. Define an interval in $\mathbb{R}$. Let I be an interval in $\mathbb{R}$ and $f:I\to\mathbb{R}$ be a function such that f' exist and is bounded on I. Prove that f is uniformly continuous on I. (1+4)

14. Suppose n$^{\text{th}}$ derivative of a function f exists finitely in a closed interval [a , a+h] . Then show that there exist a $\theta\in(0,1)$, satisfying the relation

$$f(a+h)=f(a)+hf'(a)+\frac{h^2}{2!}f''(a)+.....+\frac{h^{n-1}}{(n-1)!}f^{n-1}(a)+\frac{h^n}{n!}f^n(a+\theta h)\ .$$

15. a) Test the convergence of the series :

$$\frac{1}{2}\cdot\frac{1}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7}+\cdots$$ (4)

b) State Cauchy's condensation test . (1)

16. State Riemann rearrangement theorem on infinite series. Find a rearrangement of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ that is divergent. (1+4)

Group - B

***Answer any two questions from Question Nos. 17 to 19*:** [2×10]

17. a) On $\mathbb{R}^n$,define two operations by $\alpha\oplus\beta=\alpha-\beta$ *and* $c\cdot\alpha=-c\alpha$ for any $\alpha,\beta\in\mathbb{R}^n$ and $c\in\mathbb{R}$, where the operations on the right are the usual ones . Which of the axioms for a vector space are satisfied by $(\mathbb{R}^n,\oplus,\cdot)$? (5)

b) Show that the set of vectors $B=\{1,1+x,1+x+x^2\}$ is a basis for P_3 (vector space of dim 3) . (5)

18. Determine if the following sets are subspaces of $\mathbb{R}^3$, under the operations of addition and scalar multiplication defined component-wise . Justify your answers.

a) $W=\{(a_1,a_2,a_3)\in\mathbb{R}^3: a_1=3a_2 \text{ and } a_3=-a_2\}$. (4)

b) $W=\{(a_1,a_2,a_3)\in\mathbb{R}^3: 2a_1-7a_2+a_3=0\}$. (4)

c) $W=\{(a_1,a_2,a_3)\in\mathbb{R}^3: a_1=a_3+2\}$. (2)

19. a) Investigate for what values of λ and μ , the simultaneous equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have (i) no solutions (ii) unique solution (iii) infinite number of solutions . (4+4)

b) If A is skew symmetric , prove that $(I-A)(I+A)^{-1}$ is orthogonal . (2)

Answer <u>any two</u> questions from <u>Question No. 20 to 22</u>: [2×12]

20. a) Define i) convex hull ii) convex polyhedron iii) supporting hyperplane and separating hyperplane , with example. (8)

    b) Prove that a B.F.S to a L.P.P corresponds to an extreme point of the convex set of F.S. (4)

21. a) $x_1=2$, $x_2=3$, $x_3=1$ is a feasible solution of the equations

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

Reduce the F.S to two B.F.S . (6)

    b) Find the optimal assignment and the optimal assisgnment cost from the following cost matrix: (4)

	M_1	M_2	M_3	M_4	M_5
J_1	4	6	5	1	2
J_2	7	9	9	6	4
J_3	5	8	5	5	1
J_4	1	3	3	2	1
J_5	6	8	7	6	2

    c) What is the number of independent constraints in a balanced Transportation Problem . (2)

22. a) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b,\ x \geq 0$ corresponds to a B.F.S . (7)

    b) Solve the following Transportation problem: (5)

	D_1	D_2	D_3	D_4	a_i
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
b_j	85	35	50	45	

Answer <u>any one</u> question from <u>Question No. 23 to 24</u>: [1×6]

23. Solve the following L.P.P.by two phase method : (6)

$$\begin{aligned} &\textit{Maximize } z = 2x_1 + x_2 + x_3 \\ &\textit{Subject to } 4x_1 + 6x_2 + 3x_3 \leq 8 \\ &\qquad 3x_1 - 6x_2 - 4x_3 \leq 1 \\ &\qquad 2x_1 + 3x_2 - 5x_3 \geq 4 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

24. a) Formulate mathematically a balanced transportation problem as a L.P.P. having m origins and n destinations $(m, n \geq 2)$. (2)

    b) Find basic solutions with x_2 as one of the basic variable and discuss the nature of each solution of the following equations :

$$4x_1 + 2x_2 + 3x_3 - 8x_4 = 6$$
$$3x_1 + 5x_2 + 4x_3 - 6x_4 = 8$$

(2+2)

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